A MEASUREMENT OF TURBULENT BOUNDARY-LAYER PROFILES AND HEAT-TRANSFER COEFFICIENT AT M = 7

By
Peter P. Wegener,
Eva M. Winkler,
and
M. Sibulkin
Naval Ordnance Laboratory, White Oak, Silver Spring, Md.

AN NOL REPRINT FROM
JOURNAL OF THE AERONAUTICAL SCIENCES
Vol. 20, No. 3, 221-222, March 1953

with permission of the authors and the publisher

A Measurement of Turbulent Boundary-Layer Profiles and Heat-Transfer Coefficient at $M = 7$

Peter P. Wegener, Eva M. Winkler, and M. Sibulkin
U.S. Naval Ordnance Laboratory, White Oak, Silver Spring, Md.
October 28, 1952

Measurements in a naturally turbulent boundary layer with heat transfer to the surface were made in the continuous 12- by 12-cm. N.O.L. hypersonic tunnel.[1] The survey station was located on the centerline of one of the diverging walls of a straight-walled two-dimensional nozzle. The free-stream Mach Number (M_∞) at the survey station was 7.0, and the free-stream velocity was 3,400 ft. per sec.; the free-stream Mach Number gradient inherent in this type of nozzle was approximately $0.05M$ per in. Wind-tunnel supply pressure (p_0) and temperature (T_0) were kept constant, T_0 being 50°C. higher than the temperature needed to avoid air condensation during the expansion. Wall surface temperature (T_w) was kept constant at $T_w/T_\infty = 5.7$ by a water cooling system in the nozzle wall. The static pressures measured on the wall and at the edge of boundary layer agreed, and this value of static pressure was assumed to exist through the boundary layer. Pitot pressure was surveyed from wall to free stream, with a probe large enough to avoid errors due to slip flow effects.[2] The Mach Number determined from the Rayleigh formula is shown on Fig. 1a. (The use of the perfect gas law and $\gamma = 1.4$ implied in this procedure is valid for our test conditions.) Total temperature (T_0') distribution was surveyed with a double-shielded total temperature probe. This probe was calibrated in the free stream at about the same M and Re encountered in the boundary layer. The T_0' profile obtained after applying these calibration factors is shown in Fig. 1b. Velocity was calculated using the M data points and values of T_0' obtained from a curve faired through the measured T_0' points. A straight line was drawn between the last T_0' point and the measured value of T_w. This velocity profile is shown in Fig. 1c, which combines the results of four independent runs at identical operation conditions. A $^1/_7$-power profile is also shown in Fig. 1c for comparison. Since all the flow properties in the boundary layer are known, displacement thickness (δ^*) and momentum thickness (θ) could be computed and are indicated in Fig. 1c. The Reynolds Number based upon free-stream conditions and displacement thickness is 43,000, while an equivalent flat plate Re would be of the order of 10^7.

Other measurements show that tunnel supply temperature (T_0) is constant across the nozzle inlet. Therefore, the rate of heat transfer per unit nozzle width (Q) from the nozzle inlet to the boundary-layer station may be computed from

$$Q = c_p\rho_\infty u_\infty(T_w - T_0)\phi \tag{1}$$

$$\phi = \int_0^\infty \frac{\rho u}{\rho_\infty u_\infty}\left(\frac{T_0' - T_0}{T_w - T_0}\right) dy \tag{2}$$

Eq. (2) represents a definition of an "energy thickness" that permits the calculation of Q from the mass flow weighted defect of T_0' with respect to T_0 across the boundary layer at the survey station. On the other hand, Q may also be determined by measuring the discharge rate and temperature increase of the nozzle cooling water after a steady state of heat transfer has been established. It was found that these two values of Q were in agreement suggesting the reliability of the T_0' measurement in the boundary layer.

Measurements in the steel nozzle wall showed that the temperature dropped linearly with distance from the surface at the survey station after a steady state of heat transfer had been established. Knowing the temperature gradient (dT/dy) and the thermal conductivity (k) in the steel wall, the rate of heat transfer per unit area,

$$q = k(dT/dy) = h(T_e - T_w) \tag{3}$$

could be computed, where h is the heat-transfer coefficient. However, to determine the Stanton Number (St_∞) (also called the dimensionless heat-transfer coefficient $C_{H\infty}$),

$$St_\infty = \frac{h}{c_p\rho_\infty u_\infty} = \frac{k(dT/dy)}{c_p\rho_\infty u_\infty(T_e - T_w)} \tag{4}$$

we need to know the insulated flat-plate temperature (T_e), a quantity that we were not able to measure. Assuming the relation between our recovery factor (r) and Prandtl Number (Pr) to be $r = Pr^{1/3}$ and taking $Pr = 0.73$, we can calculate T_e and in turn obtain $St_\infty = 0.00075$ from Eq. (4) and the measured data. Using $Pr = 1$ (and thus $T_e = T_0$), we find $St_\infty = 0.00061$. However, after applying the relation[3] $c_{f\infty}/2 = St_\infty Pr^{2/3}$, we obtain $c_{f\infty} = 0.0012$ regardless of which of these Pr we choose. Knowing the local friction coefficient ($c_{f\infty}$), we calculate the shear stress at the wall (τ_w). While the heat-transfer measurement may not be too reliable, we are now able to tentatively plot our velocity profile in the u^+, y^+ parameters[4] as shown on Fig. 2. Since these parameters are of most value near the wall, the density and kinematic viscosity were based upon wall properties. The portion of the velocity profile derived from measured M but only interpolated T_0' (note actual distances from wall) is represented by a dashed line. For comparison, von Kármán's incompressible flow semilogarithmic law[4] and assumed linear velocity distribution ($u^+ = y^+$) in the laminar sublayer are shown on Fig. 2. Finally, a compressible flow profile calculated from Eq. (72) of Van Driest's analysis[5] for our M_∞ and T_w/T_∞ is shown. (Aside from other assumptions inherent in this analysis, the profile has the boundary condition $u = u_\infty$ at $y = \delta$.)

It appears from Figs. 1c and 2 that the measured velocity profile is similar to that found in low-speed flow except in the region extremely close to the wall, and this difference may be due to the lack of T_0' measurements in that region.

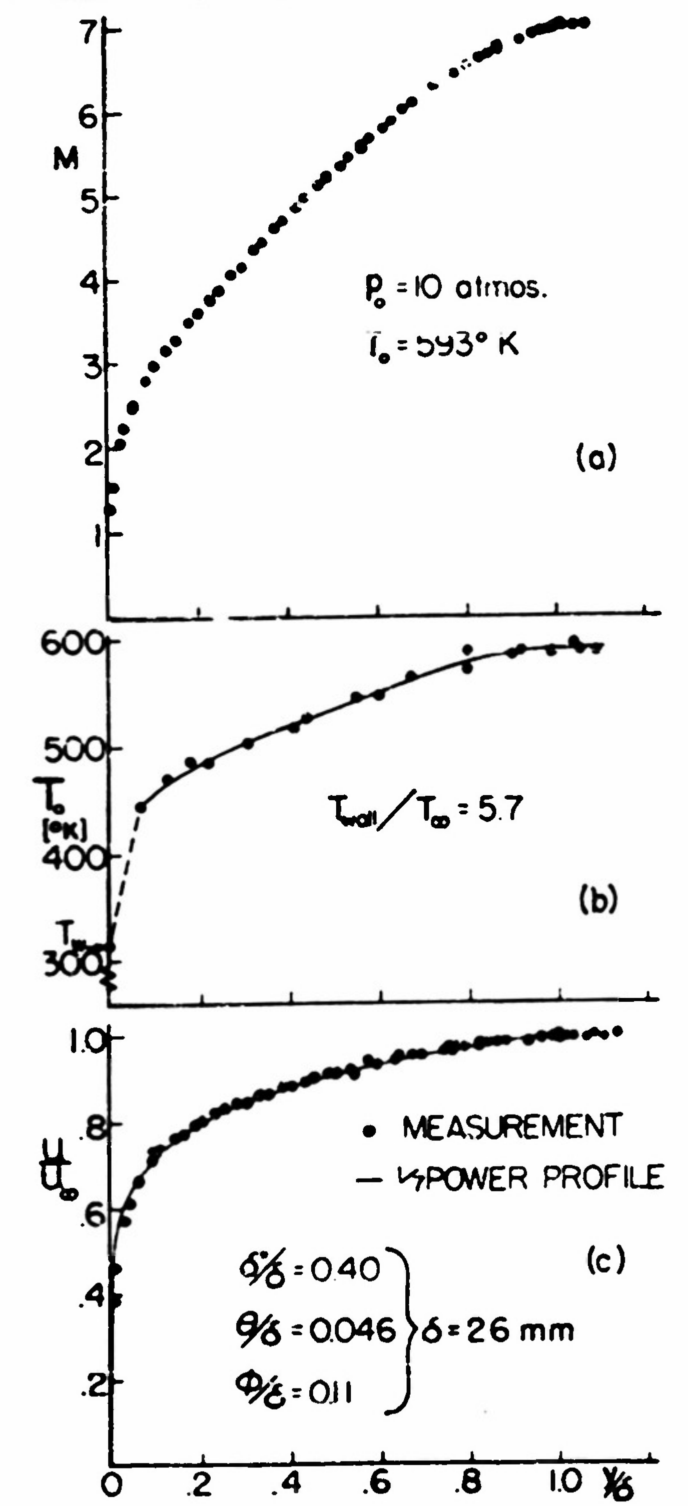

Fig. 1. Measured turbulent boundary-layer profiles with heat transfer at a free-stream Mach Number of 7.

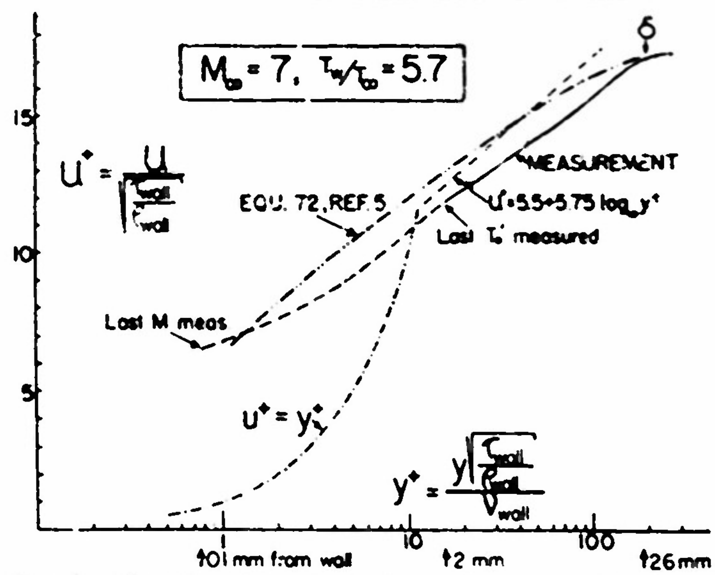

Fig. 2. The u^+, y^+ representation of the measured velocity profile at $M_\infty = 7$ and comparison with theory.

References

[1] Wegener, Peter P., *Summary of Recent Experimental Investigation in the N.O.L. Hyperballistics Wind Tunnel*, Journal of the Aeronautical Sciences, Vol. 18, No. 10, p. 665, October, 1951.

[2] Kane, E. D., and G. J. Maslach, *Impact-Pressure Interpretation in a Rarefied Gas at Supersonic Speeds*, N.A.C.A. T.N. No. 2210, October, 1950.

[3] Johnson, H. A., and M. W. Rubesin, *Aerodynamic Heating and Convective Heat Transfer—Summary of Literature Survey*, Trans. A.S.M.E., Vol. 71, No. 5, Eq. (20), p. 452, July, 1949.

[4] von Kármán, Th., *Turbulence and Skin Friction*, Journal of the Aeronautical Sciences, Vol. 1, No. 1, p. 1, January, 1934.

[5] Van Driest, E. R., *Turbulent Boundary Layer in Compressible Fluids*, Journal of the Aeronautical Sciences, Vol. 18, No. 3, p. 145, March, 1951